# Prediction of resistance and propulsion power of Ro-Ro ships

by

Hans Otto Kristensen

HOK Marineconsult ApS Hans Otto Kristensen

The Technical University of Denmark Harilaos Psaraftis

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# Resistance and propulsion power - Full-scale prediction

#### Introduction

To calculate the propulsion power of a ship, the resistance and the total propulsive efficiency have to be determined with the highest possible accuracy. As empirical methods are normally used for these calculations, it is worthwhile to know the accuracy of the different elements in the calculation procedures such that the propulsive power can be predicted in combination with an estimate of the uncertainty of the result. In the following the calculation procedures used for the present project will be described in detail and the efforts to reduce the uncertainty will be also described and discussed.

A well-known method for prediction of ship resistance is a method developed by Guldhammer and Harvald which is described in details in two publications, "Ship Resistance" [Guldhammer and Harvald 1974] and "Resistance and Propulsion of Ships" [Harvald 1983]

Following parameters are used in calculation procedure of the ship resistance R<sub>T</sub>:

Length of waterline of ship

Length between perpendiculars

B Breadth, moulded of ship

T Draught, moulded amidships (mean draught)

W<sub>I</sub> Lightship weight

D<sub>w</sub> Deadweight of ship

Δ Displacement mass of ship ( $ρ · ∇ = W_L + D_W$ )

∇ Displacement volume of ship

S The wetted surface of immersed hull

A<sub>M</sub> Immersed midship section area

A<sub>wl</sub> Area of water plane at a given draught)

D<sub>prop</sub> Propeller diameter

V Speed of ship

g gravitational constant (9.81 m/s<sup>2</sup>)

Fn Froude number (Fn =  $\frac{V}{\sqrt{g \cdot Lpp}}$ )

 $C_B$  Block coefficient,  $(C_B = \frac{\nabla}{Lpp \cdot B \cdot T})$ 

 $C_M$  Midship section coefficient ( $C_M = \frac{A_M}{R \cdot T}$ )

 $C_p$  Prismatic coefficient  $(C_P = \frac{C_B}{C_M})$ 

 $C_w$  Water plane area coefficient ( $C_w = \frac{A_{wl}}{I_{vB}}$ )

M Length displacement ratio or slenderness ratio,  $M = \frac{L}{\nabla^{1/3}}$ 

ρ Mass density of water

t Water temperature

Rn Reynolds number

The kinematic viscosity of water υ

 $C_{\mathbf{T}}$ Total resistance coefficient

Frictional resistance coefficient  $C_{\mathbf{F}}$ 

 $C_A$ Incremental resistance coefficient

Air resistance coefficient  $C_{AA}$ 

#### **Fixed values**

Design values: L, B, T, Δ, V

Calculated values (using design values): C<sub>B</sub>, C<sub>p</sub>, M, Fn, Rn

Environmental constants: Water density, temperature, kinematic viscosity

## Values assumed or calculated based on empirical methods/data for calculation of resistance and engine power

 $C_{\mathrm{T}}$ Total resistance coefficient  $C_{F}$ Frictional resistance coefficient  $C_{\mathbf{A}}$ Incremental resistance coefficient

Air resistance coefficient  $C_{AA}$  $D_{prop}$ Propeller diameter Wake fraction W

Thrust deduction fraction t Propeller efficiency,  $\eta_{o}$ 

Relative rotative efficiency  $\eta_{\mathsf{R}}$ Transmission efficiency (shaft line and gearbox losses)

Wetted surface

 $\eta_{S}$ 

#### **Total Resistance Coefficient**

The total resistance coefficient, C<sub>T</sub>, of a ship can be defined by:

$$C_{T} = C_{F} + C_{A} + C_{AA} + C_{R} = \frac{R_{T}}{\frac{1}{2} \cdot \rho \cdot S \cdot V^{2}}$$

The originally ITTC1957 method from the International Towing Tank Committee (ITTC) will used which means that the IITC 57 frictional coefficient, C<sub>F</sub>, will be used in the resistance calculations.

All parameters in the above equation will be described in the present section.

#### Wetted surface

The wetted surface is normally calculated by hydrostatic programs for calculation of the stability data for the ship. However for a quick and fairly accurate estimation of the wetted surface many different methods and formulas exist based on only few ship main dimensions, as example Mumford's formula below:

$$S = 1.025 \cdot L_{pp} \cdot (C_B \cdot B + 1.7 \cdot T) = 1.025 \cdot \left(\frac{\triangledown}{T} + 1.7 \cdot L_{pp} \cdot T\right)$$

In the present project an analysis of the wetted surface data of 52 different Ro-Ro ships (of different type as well as size) shows that the wetted surface according to the above mentioned version of Mumford's formula can be up to 15 % too small or too large. Therefore it has been analysed if the formula (i.e. the constants in the formula) can be adjusted in order to increase the accuracy of the calculation method. The results of the analysis for the wetted surface for single screw Ro-Ro ships, twin screw Ro-Ro ships and twin-skeg Ro-Ro ships are shown in Appendix B.

For a more accuarate calculation of the wetted surface, a correction taking into account the block coefficient  $C_{\text{BW}}$  (based on the water line length) has been added to the formula, as it was found that this parameter has a significant influence on the wetted surface.

The equations for the wetted surface, which have been deducted from the present analysis, are shown in the table below:

Single screw Ro-Ro ships	$S = 0.87 \cdot \left(\frac{\nabla}{T} + 2.7 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.34 \cdot C_{BW})$
Twin screw ship Ro-Ro ships with open shaft lines and twin rudders	$S = 1.21 \cdot \left(\frac{\nabla}{T} + 1.3 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.34 \cdot C_{BW})$
Twin-skeg Ro-Ro ships with two propellers and twin rudders	$S = 1.13 \cdot \left(\frac{\nabla}{T} + 1.7 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.31 \cdot C_{BW})$

The formulas for calculation of the wetted surface include the area of rudder(s) skegs and shaft lines. However any additional surfaces, S', from appendages such as bilge keels, stabilizers etc. shall be taken into account by adding the area of these surfaces to the wetted surface of the main hull separately.

If the wetted surface,  $S_1$ , is given for a given draught,  $T_1$ , the wetted surface,  $S_2$ , for another draught,  $T_2$ , can be calculated by using following formulas, which have been deducted based on an analysis of data for the different Ro-Ro ship hull forms:

Single screw Ro-Ro ships:  $S_2 = S_1 - 3.0 \cdot (T_1 - T_2) \cdot (L_{wl} + B)$ 

Conventional twin screw Ro-Ro ships:  $S_2 = S_1 - 2.5 \cdot (T_1 - T_2) \cdot (L_{wl} + B)$ 

Twin-skeg Ro-Ro ships:  $S_2 = S_1 - 3.0 \cdot (T_1 - T_2) \cdot (L_{wl} + B)$ 

Also based on a statistical analysis of three types of Ro-Ro ships following relations between  $L_{wl}$  and  $L_{pp}$  have been found:

Single screw Ro-Ro ships:  $L_{wl} = 1.01 \cdot L_{pp}$ 

Conventional twin screw Ro-Ro ships: Lwl = 1.035·Lpp

Twin-skeg Ro-Ro ships:  $L_{wi} = 1.04 \cdot L_{pp}$ 

#### Frictional resistance coefficient

The frictional resistance coefficient, C<sub>F</sub>, in accordance with the ITTC-57 formula is defined by:

$$C_F = \frac{0.075}{(\log R_n - 2)^2} = \frac{R_F}{\frac{1}{2} \cdot \rho \, S \cdot V^2}$$

where the frictional resistance,  $R_F$ , is sum of tangential stresses along the wetted surface in the direction of the motion.

 $R_{n}$  is the Reynolds number:  $R_{n} = \frac{V \cdot L_{wl}}{\upsilon}$ 

V is the ship speed in m/s and  $\upsilon$  is the kinematic viscosity of water:

$$v = ((43.4233 - 31.38 \cdot \rho) \cdot (t + 20)^{1.72 \cdot \rho - 2.202} + 4.7478 - 5.779 \cdot \rho) \cdot 10^{-6}$$

t is water temperature in degrees Celcius.

As in the original resistance calculation method by Harvald ("Ship Resistance"), it is here decided to leave out a form factor in the  $C_F$  part, but include a correction for special hull forms having U or V shape in the fore or after body, as suggested by Harvald. The influence of a bulbous bow on the resistance is included in a bulb correction, which will be described separately.

#### Incremental resistance coefficient

The frictional resistance coefficient is related to the surface roughness of the hull. However the surface roughness of the model will be different from the roughness of the ship hull. Therefore, when extrapolating to ship size, an incremental resistance coefficient  $C_A$  is added in order to include the effect of the roughness of the surface of the ship. This incremental resistance coefficient for model-ship has very often been fixed at  $C_A$  = 0.0004. However experience has shown that  $C_A$  decreases with increasing ship size and following roughness correction coefficient is proposed according to Harvald:

$\Delta = 1000  t$	$10^3 \cdot C_A = 0.6$	
$\Delta = 10000  t$	$10^3 \cdot C_A = 0.4$	
$\Delta = 100000 \text{ t}$	$10^3 \cdot C_A = 0.0$	
$\Delta = 1000000 t$	$10^3 \cdot C_A = -0.6$	

The C<sub>A</sub> values in the table can be estimated using the following expression:

$$10^3 \cdot C_A = 0.5 \cdot \log(\Delta) - 0.1 \cdot \left(\log(\Delta)\right)^2$$

#### Air resistance coefficient

Air resistance caused by the movement of the ship through the air, shall be included in the resistance calculation procedure. The air resistance X can be calculated by following formula:

$$R_{air} = X = \frac{1}{2} \cdot C_X \cdot \rho_{air} \cdot A_{VT} \cdot V^2$$

where:

C<sub>X</sub> Wind resistance coefficient

 $\begin{array}{ll} \rho_{air} & \quad \text{Density of air} \\ A_{VT} & \quad \text{Front area of ship} \end{array}$ 

The air resistance coefficient C<sub>AA</sub> is defined as follows:

$$c_{AA} = \frac{X}{\frac{1}{2} \cdot \rho_{w} \cdot V^{2} \cdot S}$$

As the ratio between air and water density is 825 the air resistance coefficient becomes:

$$C_{AA} \approx C_X \cdot \frac{A_{VT}}{825 \cdot S}$$

See Appendix A for analysis of this factor. Based on this analysis an air resistance coefficient  $C_{AA}$  value of  $0.15 \cdot 10^{-3}$  is recommended.

#### Steering resistance

It is here decided not to include a correction for added steering resistance.

#### Residual resistance coefficient - Guldhammer and Harvald

The residual resistance coefficient,  $C_R$ , is defined as the total model resistance coefficient minus the model friction resistance coefficient, i.e:

$$C_{Rm} = C_{Tm} - C_{Fm}$$

The residual resistance includes wave resistance, the viscous pressure resistance, and the additional resistance due to the form or curvature of the hull including additional drag from a large submerged transom stern.

As the residual resistance coefficient of the ship model is identical with the residual resistance coefficient of the ship,  $C_R$  is normally determined by model tests, where the resistance in model scale is measured and converted to full scale values according to methods agreed upon by the International Towing Tank Committee (ITTC) as example by using the resistance correction factors,  $C_A$  and  $C_{AA}$  as described earlier. Alternatively the residuary resistance can be predicted by empirical calculation methods, which are based on analysis of many model tests results.

One of the most well known methods has been developed by Holtrop and Mennen [Holtrop and Mennen, 1978] from the model tank in Holland (MARIN). This method is very flexible, but many details are needed as input for the calculation procedure, and the calculation model is therefore not suitable when a quick calculation procedure is needed.

In 1965 - 1974 Guldhammer and Harvald developed an empirical method ("Ship Resistance") based on an extensive analysis of many published model tests. The method depends on relatively few parameters and is used for residual resistance prediction in the present analyses. Harvald presents curves (see Appendix G) for  $C_R$  ( $C_{R,Diagram}$ ) as function of three parameters: 1) The length-displacement ratio, 2) the prismatic coefficient and finally 3) the Froude number. The coefficient is given without correction for hull form, bulbous bow or position of LCB and appendages such as shaft lines and shaft brackets. Harvald gives additional corrections for these parameters.

The residual resistance coefficient curves must be corrected for:

- Position of LCB (ΔC<sub>R,LCB</sub>)
- Shape / hull form (ΔC<sub>R.form</sub>)
- B/T deviation from 2.5 ( $C_R$  curves are all given a breadth-draft ratio equal 2.5) ( $\Delta C_{R,B/T\neq2.5}$ )
- Bulbous bow shape and size ( $\Delta C_{R,bulb}$ )

$$C_R = C_{R,Diagram} + \Delta C_{R,B/T \neq 2.5} + \Delta C_{R,LCB} + \Delta C_{R,form} + \Delta C_{R,bulb}$$

A proposal for corrections for LCB not placed amidships in the vessel is given. Harvald allows only LCB forward of amidships and the correction will always be positive, which gives an increased resistance.

→ In the present analysis the LCB correction will be ignored

The correction for both the hull form and the B/T correction are used as described by Harvald. These factors are assumed not to have changed since the method was developed by Harvald; the correction must be the same disregarding age of vessel.

- → Correction of form and B/T is in the present project taken as Harvald recommends: No correction for B/T equal 2.5, else  $\Delta C_{R,B/T\neq2.5}=0.16\cdot\left(\frac{B}{T}-2.5\right)\cdot10^{-3}$
- → Hull form

A hull shape correction to  $C_R$  is applied if the aft or fore body is either extremely U og V shaped

Fore body Extreme U:  $-0.1 \cdot 10^{-3}$  Extreme V:  $+0.1 \cdot 10^{-3}$  After body Extreme U:  $+0.1 \cdot 10^{-3}$  Extreme V:  $-0.1 \cdot 10^{-3}$ 

Bulbous bow forms have been optimised and bulbs developed in the recent years can reduce the resistance quite considerably. Earlier non-projecting bulbous bows decreased resistance at best by some 5-10%. Modern bulbs can decrease resistance by up to 15-20% [Schneekluth and Bertram 1998]. See also Fig. C4 in Appendix C.

→ New analyses and equations for bulbous bow corrections will be included in the present analyses.

As described earlier the curves for  $C_R$  are given as function of the three parameters: The length-displacement ratio (M), the prismatic coefficient ( $C_P$ ) and finally the Froude number (Fn).

• M: Length-displacement ratio  $M = \frac{L_{WL}}{\nabla^{1/3}}$ 

•  $C_P$ : Prismatic coefficient  $C_P = \frac{C_B}{C_M}$ 

• Fn: Froude number

#### Midship section coefficient

The midship section coefficient,  $C_M$ , is defined as the immersed midship section area,  $A_M$ , divided by the rectangular area of the breadth and draught, i.e.  $C_M = A_M/(B \cdot T)$ .

 $C_{\text{M}}$  has been analyzed for 64 Ro-Ro ships and  $C_{\text{M}}$  is plotted as function of the block coefficient,  $C_{\text{B}}$  in Fig. 1, where the relation between  $C_{\text{M}}$  and  $C_{\text{B}}$  is shown as follows:

$$C_{\rm M} = 0.38 - 1.25 \cdot C_{\rm B}^2 + 1.725 \cdot C_{\rm B}$$
 and  $C_{\rm M} = 0.975$  for  $C_{\rm B} > 0.7$ 

The midship section coefficient,  $C_M$ , will slightly decrease for decreasing draft according to following formula:

$$C_{M1} = 1 - \frac{T_0}{T_1} \cdot (1 - C_{M0})$$

where:

 $C_{M0}$  is the midship coefficient at draught  $T_0$  and  $C_{M1}$  is the midship coefficient at draught  $T_1$ 

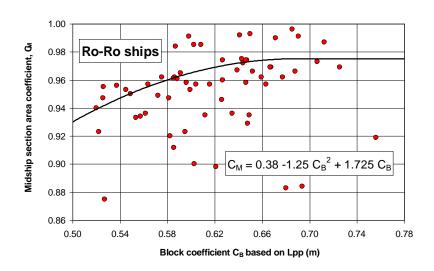


Fig. 1 Midship section coefficient, C<sub>M</sub>, for 64 Ro-Ro ships

#### Prismatic coefficient, C<sub>P</sub>, and length displacement ratio, M, for Ro-Ro ships

Fig. 2 shows that M and C<sub>P</sub> vary within following limits for Ro-Ro ships:

M: 
$$4.8 - 8.3$$
  $C_P$ :  $0.55 - 0.78$ 

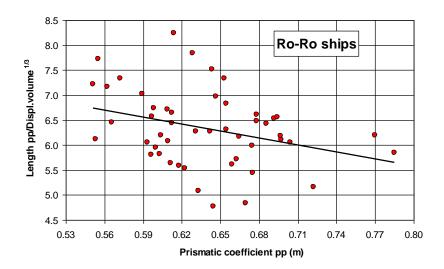


Fig. 2 Relation between prismatic coefficient and length displacement ratio

#### Froude number

The resistance and the associated resistance coefficients depend on the speed in non-dimensional form defined as the Froude number, Fn as follows:

$$Fn = \frac{V}{\sqrt{g \cdot L}}$$
 where

V is the ship speed in m/s g is the acceleration due to gravity (9.81 m/s²) L is the ship length

## Formulas for calculation of the standard residuary resistance coefficient

By an extensive regression analysis of the original Cr curves (shown in Appendix G) following expressions have been developed by Guldhammer in 1978:

$$C_R = f(M, C_P, Fn)$$

$$10^3 \cdot C_R = E + G + H + K$$

where:

$$E = (A_o + 1.5 \cdot Fn^{1.8} + A_1 \cdot Fn^{N_1}) \cdot \left(0.98 + \frac{2.5}{(M-2)^4}\right) + (M-5)^4 \cdot (Fn - 0.1)^4$$

$$A_o = 1.35 - 0.23 \cdot M + 0.012 \cdot M^2$$

$$A_1 = 0.0011 \cdot M^{9.1}$$

$$N_1 = 2 \cdot M - 3.7$$

$$G = \frac{B_1 \cdot B_2}{B_3}$$

$$B_1 = 7 - 0.09 \cdot M^2$$

$$B_2 = (5 \cdot C_P - 2.5)^2$$

$$B_3 = (600 \cdot (Fn - 0.315)^2 + 1)^{1.5}$$

$$H = EXP(80 \cdot (Fn - (0.04 + 0.59 \cdot C_P) - 0.015 \cdot (M - 5)))$$

$$K = 180 \cdot Fn^{3.7} \cdot EXP(20 \cdot C_P - 16)$$

The formula for Cr is valid for Fn <= 0.33

#### **Bulbous bow correction for Ro-Ro ships**

In the method by Guldhammer and Harvald it is assumed that the ship has a standard non bulbous bow. The method therefore includes corrections for a bulbous bow having a cross section area of at least 10 % of the midship section area of the ship. There has been written much about the influence of a bulbous bow on the ship resistance. Many details have an influence, as example the transverse and longitudinal shape of a bulbous bow including its height compared to the actual operational draught.

The bulb correction might, as  $C_R$ , be function of three parameters:

- 1) The length-displacement ratio (M)
- 2) The prismatic coefficient (C<sub>P</sub>) and
- 3) The Froude number (Fn).

However for a given condition/draught the wave pattern and therefore the residual resistance varies mainly with the speed when the ship is operated at the design draught, i.e. the draught where the bulbous bow is just submerged. Around this draught the bulbous bow correction will therefore mainly be a function of the Froude number, which is assumed in the present analysis.

$$\Delta C_{R.bulb} = \Delta C_{R.bulb}(Fn)$$

As just mentioned the bulb correction will also be draft and trim dependent, but this dependency can be very complex. In general the bulb correction will reach its highest value, when the bulbous bow is just slightly submerged at its design draught. When the waterline is below the upper surface of the bulbous bow the positive influence decreases and in the worst case completely disappears.

In the present project, the bulb correction is determined by analysis of model tests results for 34 Ro-Ro ships having a bulbous bow. The total resistance coefficient of each individual ship has been calculated by Guldhammer and Harvalds method without any corrections for bulbous bow. Subtracting this value from the total resistance coefficient found by model tests gives the bulbous bow correction which is needed for updating of the "Ship Resistance" method. See Appendix C.

For Ro-Ro ships with conventional hull form (either single or twin screw hull form) the correction thus found can be approximated by following formula:

$$10^3 \cdot \Delta C_{R,bulb} = -0.2 - 1.1 \cdot Fn$$
 (see Fig. 3)

For Ro-Ro ships with so called twin-skeg hull form (twin screw propulsion) the bulb correction thus found can be approximated by following formula:

$$10^3 \cdot \Delta C_{R.bulb} = 0.52 - 2.6 \cdot Fn$$
 (see Fig. 4)

For both hull forms the Froude number is based on the waterline length of the ship.

For conventional Ro-Ro hull forms the bulb correction will be negative for the whole range of Froude numbers, meaning that the bulb will decrease the total resistance. For twin-skeg vessels the bulb correction is smaller and a bit more complex, which is most probably due to the typical stern shape of twin-skeg hull forms, with a large transom stern. The transom stern often creates a large stern wave, which has a negative influence on the residuary resistance of these vessels.

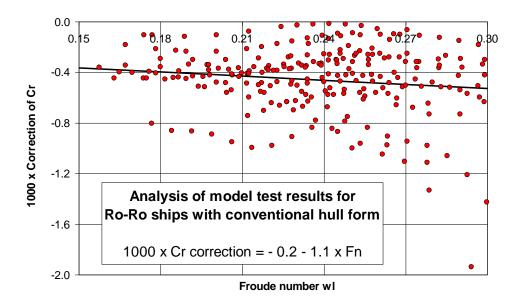


Fig. 3 The bulb correction for the residuary resistance coefficient for conventional hull forms

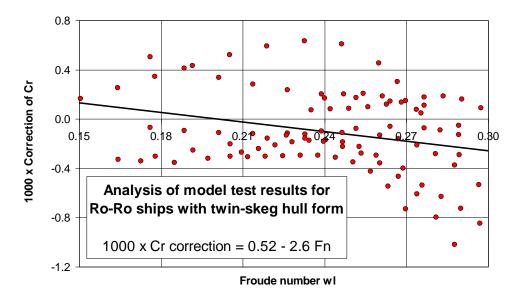


Fig. 4 The bulb correction for the residuary resistance coefficient for twin-skeg hull forms

#### **Appendage Cr correction**

For a single screw ship the added resistance from the single rudder is assumed included in the  $C_R$  value. However for twin screw ships with conventional hull forms, with open shafts and shaft brackets, these will induce added resistance, which can be treated as a separate Cr correction. Analysis of model tests (not presented in this report) show that the average Cr appendage correction for a typical well-designed twin screw propeller shaft system is appr.  $0.3 \cdot 10^{-3}$ . This value has been used in the present analysis of all the model tests for conventional twin screw ships and of that reason it is also recommended to be used in the calculation of the total resistance coefficient.

#### **Total ship resistance**

$$R_T = \frac{1}{2} \cdot C_T \cdot \rho \cdot S \cdot V^2$$

#### **Effective power**

$$P_E = R_T \cdot V$$

#### Service allowance

The service allowance is used for determination of the installed main engine power, which means that it shall be determined based on the expected service area. Harvald suggests following service allowances:

North Atlanctic route, westbound	25 – 35 %
North Atlantic, eastbound	20 – 25 %
Europe Australia	20 – 25 %
Europe – Eastern Asia	20 – 25 %

The Pacific routes 20 – 30 %

The above figures are only rough figures, which can be used for guidance. For more accurate predictions, the size of the ship shall be taken into account, as the service allowance will be relatively higher for small ships compared to large ships. Furthermore the hull form will also have an influence on the necessary service allowance. The more slender hull form, the less service allowance is needed.

$$P_E$$
service =  $R_T \cdot V \cdot (1 + \frac{\text{service allowance in \%}}{100})$ 

#### **Propulsive efficiencies**

## Total efficiency

$$\begin{split} \eta_{T} &= \eta_{H} \cdot \eta_{O} \cdot \eta_{R} \cdot \eta_{S} \\ \eta_{T} & \text{Total efficiency} \\ \eta_{H} & \text{Hull efficiency} \end{split}$$

 $\eta_o$  Propeller in open water condition

 $\eta_R$  Relative rotative efficiency

η<sub>S</sub> Transmission efficiency (shaft line and gearbox)

#### Hull efficiency

 $\eta_H$  The hull efficiency is a function of the wake fraction, w, and the thrust deduction fraction, t, [Harvald 1983]

$$\eta_H = \frac{1-t}{1-w}$$

Wake fraction:  $w = w_1 \left(\frac{B}{L}, C_B\right) + w_2 (form, C_B) + w_3 \left(\frac{D_{prop}}{L}\right)$ 

Thrust deduction fraction:  $t = t_1 \left( \frac{B}{L}, C_B \right) + t_2 (form) + t_3 \left( \frac{D_{prop}}{L} \right)$ 

For normal N-shaped hull forms,  $w_2$  and  $t_2$  will be equal 0, which means that both the wake fraction and the thrust deduction is a function of the breadth-length ratio, the ratio of the propeller diameter and the length and finally the block coefficient.

The form in the aft body  $(F_a)$  can be described by factors: [-2, 0, +2], negative values for U-shape, positive for V-shape and zero for N-shaped hull form.

The approximations given by Harvald are used in the present work. In [Harvald 1983] are all values given in diagrams. These values are approximated by simple regression formulas as follows.

The wake fraction for single screw ships:

$$\begin{aligned} w &= w_1 + w_2 + w_3 \\ w_1 &= a + \frac{b}{c \cdot (0.98 - C_B)^3 + 1} \\ w_2 &= \frac{0.025 \cdot F_a}{100 \cdot (C_B - 0.7)^2 + 1} \\ w_3 &= -0.18 + \frac{0.00756}{\frac{D_{Prop}}{L} + 0.002} \text{ and } w_3 \le 0.1, \\ a &= \frac{0.1 \cdot B}{L} + 0.149 \\ b &= \frac{0.05 \cdot B}{L} + 0.449 \\ c &= 585 - \frac{5027 \cdot B}{L} + 11700 \cdot \left(\frac{B}{L}\right)^2 \end{aligned}$$

For trial trip conditions with clean hull the wake fraction shall be reduced by 30% for single screw ships. For twin screw vessels no reduction is to be applied.

The trust deduction fraction for single screw ships:

$$\begin{split} t &= t_1 + t_2 + t_3 \\ t_1 &= d + \frac{e}{f \cdot (0.98 - C_B)^3 + 1} \\ t_2 &= -0.01 \cdot F_a \\ t_3 &= 2 \cdot \left(\frac{D_{Prop}}{L} - 0.04\right) \\ d &= \frac{0.625 \cdot B}{L} + 0.08 \\ e &= 0.165 - \frac{0.25 \cdot B}{L} \\ f &= 825 - \frac{8060 \cdot B}{L} + 20300 \cdot \left(\frac{B}{L}\right)^2 \end{split}$$

For <u>conventional twin screw ships</u> the wake fraction and thrust deduction fraction are calculated according to formulas based on Harvald [Harvald 1983, Figure 6.5.8]:

$$w = 1.133 \cdot C_B^2 - 0.797 \cdot C_B + 0.215$$
$$t = 0.0665 + 0.62833 \cdot w$$

For <u>twin-skeg ships</u> the wake fraction will be higher due to the skeg in front of each propeller. Based on analysis of 12 model test results with twin-skeg Ro-Ro ships (shown in Appendix E) following equations have been established for calculation of the wake fraction and the thrust deduction fraction of twin-skeg vessels:

$$w = 0.7 \cdot C_B - 0.2$$

$$t = 0.19$$

## Propeller diameter

 $D_{\mathrm{prop}}$  is the propeller diameter. If not known the following approximations can be used to calculate  $D_{\mathrm{prop}}$  as function of the maximum draught (see Appendix D for statistical analysis):

Single screw Ro-Ro ships (cargo and pass):  $D_{prop} = 0.56 \cdot max. draught + 1.07$ 

Twin screw Ro-Ro cargo ships:  $D_{prop} = 0.71 \cdot max. draught - 0.26$ 

Twin screw Ro-Ro passenger ships:  $D_{prop} = 0.85 \cdot max. draught - 0.69$ 

#### Propeller efficiency

By expressing the open water efficiency as function of the thrust loading coefficient, it is possible to obtain a relatively accurate efficiency without a detailed propeller optimization procedure. As the thrust loading depends on the propeller diameter and the resistance, these two parameters are automatically included in the efficiency calculation.

 $\eta_o$  In Breslin and Andersen [1994] are presented curves for efficiencies of various propulsion devices. The efficiency is presented as function of the thrust loading coefficient  $C_{Th}$ .

The trust loading coefficient:

$$\begin{split} C_{Th} &= \frac{T}{\frac{1}{2} \cdot \rho \cdot A_{disk} \cdot V_A{}^2} \qquad \text{and} \qquad C_{Th} = \frac{8}{\pi} \cdot \frac{R}{(1-t) \cdot \rho \cdot \left(V_A \cdot D_{prop}\right)^2} \qquad \text{as} \\ C_{Th} &= \frac{8}{\pi} \cdot \frac{K_T}{J^2} \qquad \qquad J = \frac{V_A}{n \cdot D} \qquad \qquad K_T = \frac{R}{(1-t) \cdot \rho \cdot n^2 \cdot D_{prop}^4} \end{split}$$

 $R = (1 - t) \cdot T \qquad V_A = (1 - w) \cdot V$ 

Breslin and Andersen [1994] show curves for approximated values of  $\eta_o$  for the conventional Wageningen B – series propellers (Fig. 8 in this section). The values taken from this curve will here be denoted as  $\eta_{o,Wag}$ 

As the propeller efficiency is primary a function of the thrust loading coefficient  $C_{Th}$ , it is the intention is to determine a function, f, so  $\eta_{o,Wag} = \eta_{o \ ideal} \cdot f(C_{Th})$ 

where  $\eta_{o ideal}$  is the co-called ideal efficiency defined by:

$$\eta_{o \; ideal} = \frac{2}{1 + \sqrt{\frac{T}{\frac{1}{2} \cdot \rho \cdot A_{disk} \cdot {V_A}^2} + 1}} = \frac{2}{1 + \sqrt{C_{Th} + 1}}$$

When dividing  $\eta_{o,Wag}$  with  $\eta_{o\,ideal}$  it is found that  $f(C_{Th})$  can be expressed by a linear function:  $f(C_{Th}) = 0.81 - 0.014 \cdot C_{Th}$  however not lower than 0.65 resulting in following equation:

$$\eta_{o,Wag} = \frac{2}{1 + \sqrt{C_{Th} + 1}} \text{ Max}(0.65; (0.81 - 0.014 \cdot C_{Th}))$$

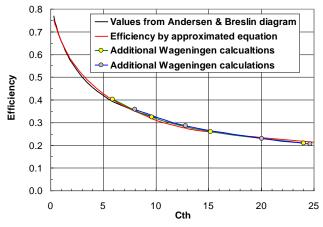
In Fig. 5 are shown comparisons between the Wageningen efficiency values form Andersen and Breslin (Fig. 8) and the above mentioned approximate equation and some additional results from Wageningen B-series calculations. These additional calculated results were prepared to cover a larger  $\mathcal{C}_{Th}$  range than obtained from Andersen and Breslin.

The efficiency calculated by the approximated propeller efficiency equation is compared with some open water efficiencies found from model tests with different ship types (Fig. 6). From this comparison it is observed that the model tests results are 3-5% lower than the approximated Wageningen efficiency.

Experience (by model tanks and propeller manufacturers) from comparisons of efficiencies from model tests with full-scale efficiencies shows that model test values are normally  $3-5\,\%$  lower than full-scale values. This means that the propeller efficiency obtained by the above mentioned expression represents the full scale efficiency.

In the efficiency diagram by Andersen and Breslin (Fig. 8) is also shown an efficiency curve for a ducted propeller solution (denoted "Kort nozzle"). Using the same principles as for the Wageningen propeller curves following equation has been derived for the ducted propeller efficiency  $\eta_{o,nozzle}$ :

$$\eta_{o,nozzle} = \eta_{o ideal} \cdot g(C_{Th})$$



0.70
0.66
0.62
0.58
0.54
0.50
0.46
0.0
0.5 1.0
1.5 2.0 2.5 3.0 3.5
Cth

Fig. 5 Efficiencies for a Wageningen B-series propeller based on Andersen and Breslin and numerical approximation

Fig. 6 Propeller Wageningen B series efficiencies from Andersen and Breslin compared with efficiencies obtained from model tests

Up to a  $C_{Th}$  value of 7 the function  $g(C_{Th})$  can be approximated by a forth degree polynomial of  $C_{Th}$ , as shown below:

$$g = 0.59 + 0.177 \cdot C_{Th} - 0.0462 \cdot C_{Th}^2 + 0.00518 \cdot C_{Th}^3 - 0.000205 \cdot C_{Th}^4$$

for 
$$C_{Th} < 7$$
 and for  $C_{Th} \ge 7$ :  $g = 0.85$ 

In Fig. 7 are shown comparisons between the nozzle efficiency values from Andersen and Breslin and the above mentioned approximate equation for a nozzle propeller.

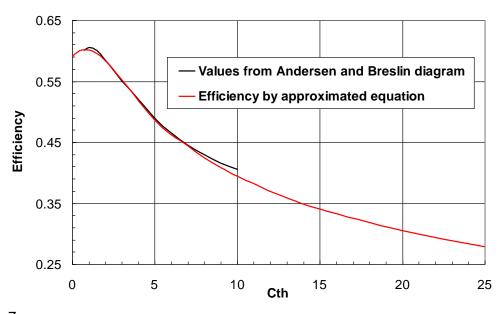


Fig. 7 Efficiencies for a nozzle propeller based on Andersen and Breslin and numerical approximation. Normally  $C_{\text{Th}}$  is less than 10, but the efficiency approximation has been extended in order to cover more extreme bollard pull conditions where  $C_{\text{Th}}$  is higher than 10.

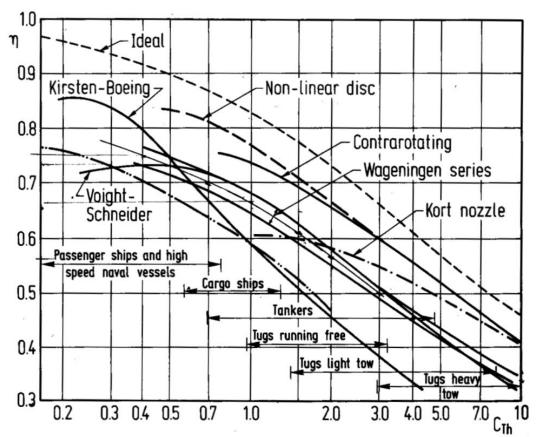


Fig. 8 Efficiencies of various propulsion devices and C<sub>Th</sub> for different ship types [Andersen and Breslin]

#### Relative rotative efficiency and shaft efficiency

 $η_{o}$ ,  $η_{R}$  Behind propeller efficiency,  $η_{B}$ ,is defined as:  $η_{B} = η_{O} \cdot η_{R} \sim η_{O}$  where the relative rotative efficiency  $η_{R}$  in average is close to one. An analysis of model test results shows following results for Ro-Ro ships (see Fig. 9 and 10) with following average values:

 $\begin{array}{ll} \mbox{Conventional hull forms:} & \eta_R = 1.01 \\ \mbox{Twin-skeg hull forms:} & \eta_R = 1.03 \\ \end{array}$ 

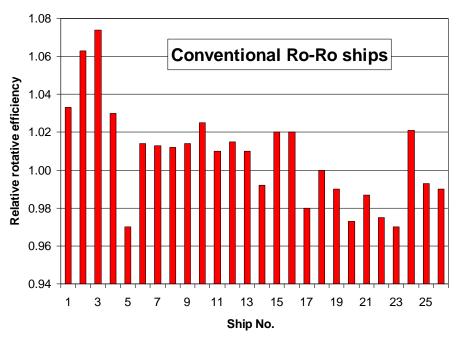


Fig. 9 Relative rotative efficiency found by model tests for conventional Ro-Ro ships

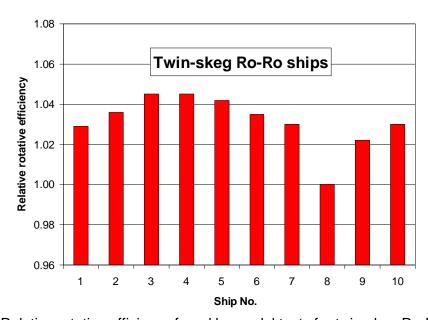


Fig. 10 Relative rotative efficiency found by model tests for twin-skeg Ro-Ro ships

ης

The transmission efficiency  $\eta_s$  is the ratio between the mechanical power needed for propulsion, i.e. driving the propeller(s) and the power delivered directly to the propeller(s).  $\eta_s$  is therefore a measure of the mechanical and electrical losses between the prime mover(s) and the propeller(s). The transmission losses depend of different factors such as the propeller shaft length, number of bearings and possible gearboxes in the shaft line. If the propeller is driven by an electric motor as a part of a diesel-electric propulsion system additional losses in the diesel-electrical power conversion shall be taken into account when  $\eta_s$  has to be determined.

For a shaft line, where the propeller is directly coupled to a diesel engine,  $\eta_s$  is approximately 0.98, while  $\eta_s$  is 0.96 – 0.97 for a shaft system where a gearbox is

included in the propulsion line. For a diesel-electric propulsion system the total mechanical and electrical transmission losses is approximately 10 %, resulting in a  $\eta_c$  value of 0.9.

### Propulsion power, PP

$$P_{P} = \frac{P_{E}}{\eta_{T}}$$

#### Test calculations

After the determination of all the empirical formulas for calculation of:

- Resistance coefficients and associated corrections
- The wetted surface
- Wake and thrust deduction fraction for calculation of hull efficiency
- Propeller and relative rotative efficiency

the propulsive power has been calculated for all the ships, for which the model test results are available and which are used for the development of the empirical formulas as described in this report.

The results of the power calculations for each individual ship are shown in Appendix F.

It is observed that for most of the ships there is a good agreement between the power prediction based on model tests and the described empirical method. The deviations between the model test based values and the values based on the described calculation method are summarized in Fig. 10 and 11 respective for conventional Ro-Ro ship hull forms and for twin-skeg hull forms.

For the conventional hull forms it is seen that the maximum deviation for 4 of the 25 ships is plus/minus approximately 20 %, while the maximum deviation for 3 of the 10 twin skeg ships is 20 – 30 %. For the remaining ships the maximum deviation is in the order of plus/minus 10 %.

In average the propulsion power found by model tests for conventional hull form is 2 % higher than the power found by the empirical method. An analysis of the propeller efficiency found by model tests shows that this is in average 4 % lower than the open water efficiency calculated by the empirical formulas (see also previous remarks about Fig. 6). For the twin skeg ships the propulsion power found by model tests is in average 6 % higher than the power found by the empirical method. An analysis of the propeller efficiency found by model tests shows that for twin-skeg ships the open water efficiency is in average 6 % lower than the open water efficiency calculated by the empirical formulas. Taking into account that the real open water efficiency will most probably be 4 - 6 % higher, means that the empirical power prediction method in average gives quite reliable results. The deviations might often be due to very good hull lines and in other cases bad hull lines which could be further improved by carful redesign of the hull shape.

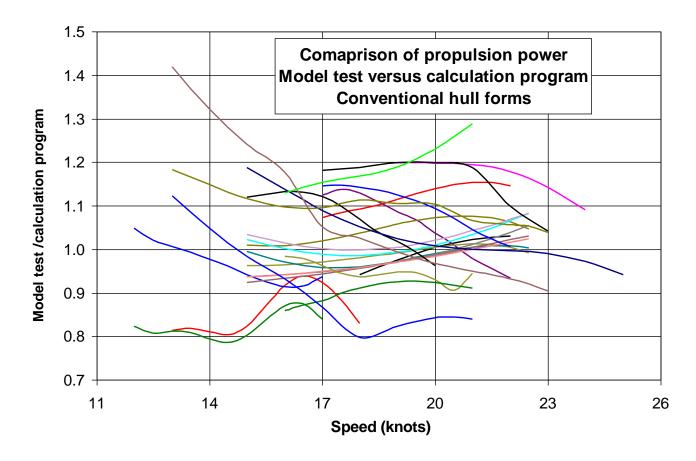


Fig. 10 Ratio between model test based power predictions and empirical calculated power predictions for conventional Ro-Ro ship hull forms.

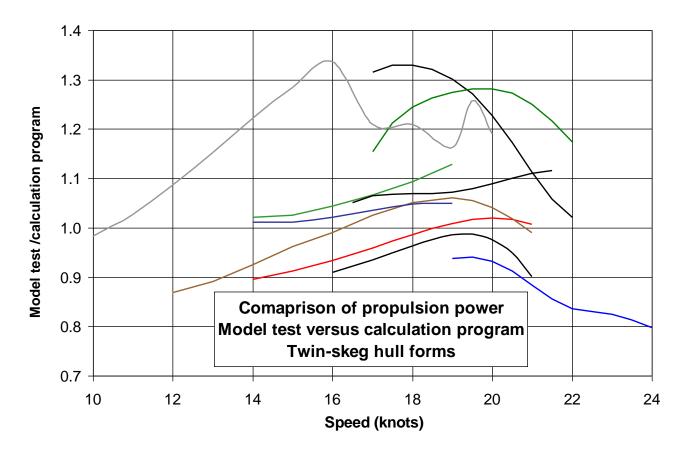


Fig. 11 Ratio between model test based power predictions and empirical calculated power predictions for twin-skeg Ro-Ro ship hull forms.

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# Appendix A – Air resistance

The axial wind force coefficient:  $C_X = \frac{X}{\frac{1}{2} \cdot \rho_{air} \cdot V^2 \cdot A_{VT}}$ 

The air resistance coefficient:  $C_{AA} = \frac{x}{\frac{1}{2} \cdot \rho_{w} \cdot V^{2} \cdot S}$ 

The relation between  $C_{AA}$  and  $C_x$ :  $C_{AA} = C_X \cdot \frac{\rho_{air}}{\rho_w} \cdot \frac{A_{VT}}{S} \approx C_X \cdot \frac{A_{VT}}{825 \cdot S}$ 

The value of  $C_x$  [Blendermann 1986]: 0.80

Wetted surface: Se Appendix B.

Estimation of front area  $A_{VT}$ :  $A_{VT} = B \cdot (D - T + 3 \cdot N)$ 

N is the number if tiers above the upper deck assuming an average height of 3 m for each tier. N depends on the length of the ship as follows:

$$N = 0.2 + 0.03 L_{pp}$$

Based on statistical data of 229 Ro-Ro ships the  $C_{AA}$  value has been calculated using the above mentioned formulas, and the results of these calculations are shown in fig. A1. Based on these results,  $C_{AA}$  has been assumed to be  $0.15 \cdot 10^3$ , as a slightly conservative assumption.

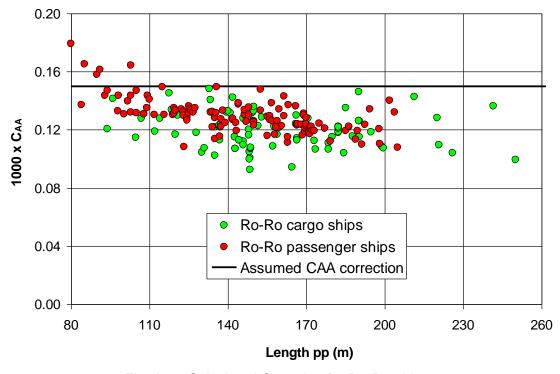


Fig. A1 Calculated C<sub>AA</sub> value for Ro-Ro ships

# Appendix B - Wetted surface of Ro-Ro ships

The equation used for calculation of the wetted surface in the present project is Mumfords formula according to [Harvald 1983, p. 131]:

$$S = 1.025 \cdot L_{pp} \cdot (C_B \cdot B + 1.7 \cdot T) = 1.025 \cdot \left(\frac{V}{T} + 1.7 \cdot L_{pp} \cdot T\right)$$

An analysis of wetted surface data of 52 different Ro-Ro ships (of different type as well as size) shows that the wetted surface according to the above mentioned version of Mumford's formula can be up to 15 % too small or too high (Fig. B6 and B7). Therefore it has been analysed if the formula can be adjusted to increase the accuracy.

Analysis of ship geometry data has shown that the wetted surface can be calculated according to following modified Mumford formulas:

$$S = X \cdot \left(\frac{\nabla}{T} + 2.7 \cdot L_{wl} \cdot T\right)$$
 for single screw Ro-Ro ships

$$S = X \cdot \left( \frac{\overline{v}}{T} + 1.3 \cdot L_{wl} \cdot T \right)$$
 for twin screw Ro-Ro ships

$$S = X \cdot \left( \frac{\triangledown}{T} + 1.7 \cdot L_{wl} \cdot T \right)$$
 for twin-skeg Ro-Ro ships

The X- value for the three different ships types are show in Fig. B1

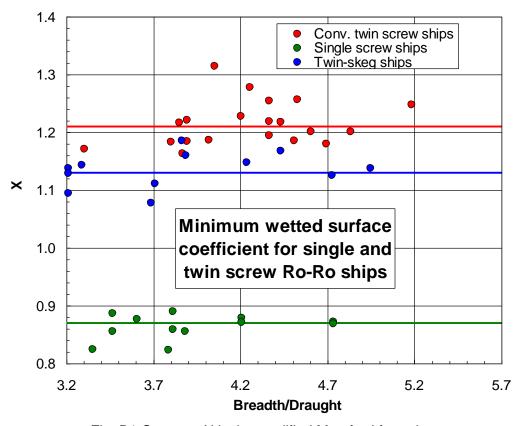


Fig. B1 Constant X is the modified Mumford formula

Using the modified Mumford formulas increases the accuracy of calculation of the wetted surface. However a further analysis reveals that the block coefficient also has an influence on the wetted surface, which can be seen by comparing the actual wetted surface with the wetted surface calculated according to the revised Mumford formula.

The results of this comparison are shown on Fig. B2 – B4. Based on the correction factors following equations for calculation of the wetted surface have been deducted:

Single screw Ro-Ro ships	$S = 0.87 \cdot \left(\frac{\nabla}{T} + 2.7 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.34 \cdot C_{BW})$
Twin screw ship Ro-Ro ships with open shaft lines and twin rudders	$S = 1.21 \cdot \left(\frac{\nabla}{T} + 1.3 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.34 \cdot C_{BW})$
Twin-skeg Ro-Ro ships with two propellers and twin rudders	$S = 1.13 \cdot \left(\frac{\nabla}{T} + 1.7 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.31 \cdot C_{BW})$

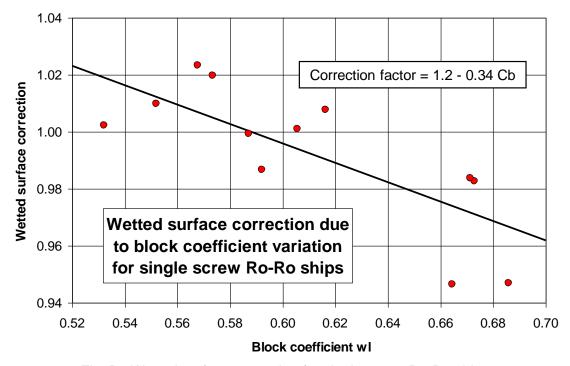


Fig. B2 Wetted surface correction for single screw Ro-Ro ships

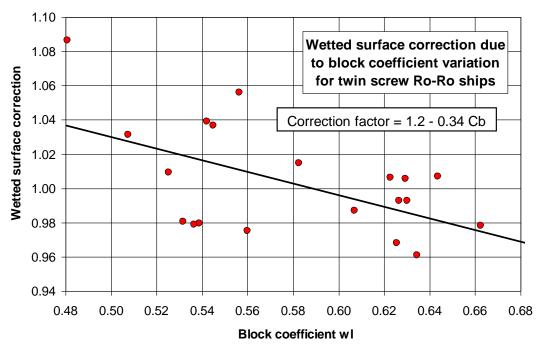


Fig. B3 Wetted surface correction for twin screw Ro-Ro ships

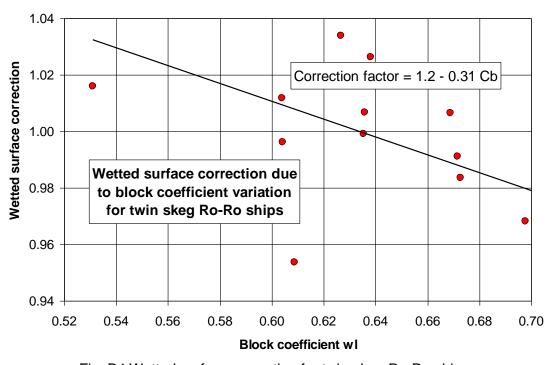


Fig. B4 Wetted surface correction for twin-skeg Ro-Ro ships

Comparisons of the wetted surface using the different formulas with the actual wetted surface are shown in Fig. B5 - B7. It is seen that the modified versions of Mumfords formula increases the accuracy considerable - with the smallest difference using the formula with block coefficient correction. It is seen that the difference is less than 3 % for 86 % of the single screw ships and 69 % of the conventional twin screw ships. For the twin-skeg ships the accuracy is even better as the difference is below 2 % for 79 % of these ships.

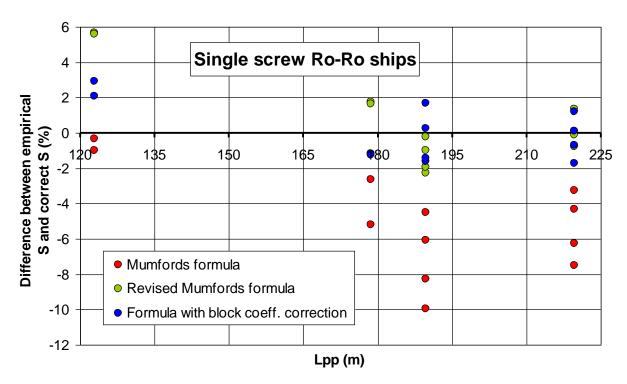


Fig. B5 Difference between the wetted surface according to different versions of Mumfords formula and the actual wetted surface for single screw Ro-Ro ships

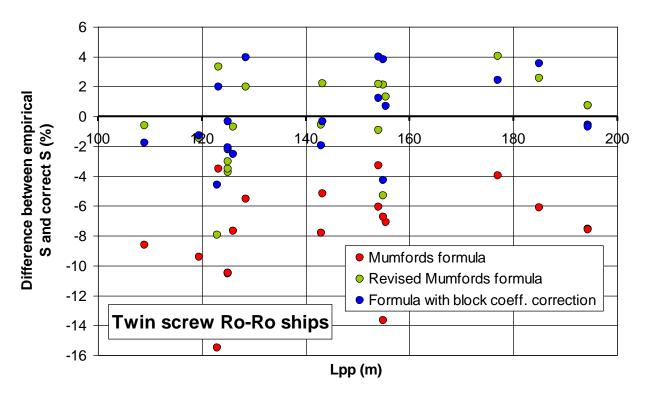


Fig. B6 Difference between the wetted surface according to different versions of Mumfords formula and the actual wetted surface for conventional twin screw Ro-Ro ships

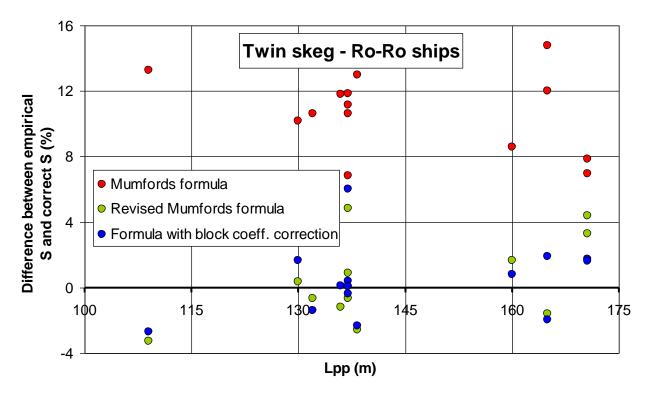


Fig. B7 Difference between the wetted surface according to different versions of Mumfords formula and the actual wetted surface for twin-skeg Ro-Ro ships

Table B1 Average difference in % between the wetted surface according to different versions of Mumfords formula and the actual wetted surface for Ro-Ro ships

Ship type	Original Mumford formula	Modified Mumford formula	Modified Mumford formula with block coefficient correction
Single screw ship	4.94	1.86	1.34
Conventional twin screw ship	5.80	2.80	2.53
Twin-skeg ship	10.68	2.15	1.65

# Appendix C – Bulbous bow resistance correction for Ro-Ro ships

In the present project, the bulb correction is determined by analysis of model tests results for 34 Ro-Ro ships having a bulbous bow. The total resistance coefficient of each individual ship has been calculated by Guldhammer and Harvalds method without any corrections for bulbous bow. Subtracting this value from the total resistance coefficient found by model tests gives the bulbous bow correction which is needed for updating of the "Ship Resistance" method.

The results of this analysis for 382 model test values for ships with a bulbous bow are shown in figure C1. The figure show positive influence of the bow for increasing Froude number.

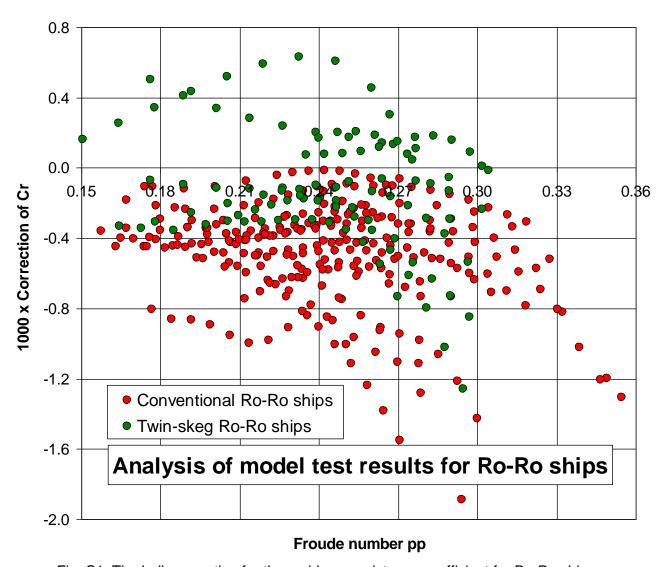


Fig. C1 The bulb correction for the residuary resistance coefficient for Ro-Ro ships

It is seen that for conventional Ro-Ro hull forms the bulb correction will be negative for the whole range of Froude numbers, meaning that the bulb will decrease the total resistance. For twin-skeg vessels the bulb correction is smaller which is most probably due to the typical stern shape of twin-skeg hull forms, with a large transom stern which can create a large stern wave, which has a negative influence on the residuary resistance of these vessels.

It is seen that for Froude numbers above 0.3, the bulbous bow correction is rather large which is considered to have a too large influence on the establishment of the bulb correction, of which reason correction values for Froude numbers above 0.30 have been disregarded in the final analysis which are shown in Fig. C2 and C3.

For Ro-Ro ships with conventional hull form (either single or twin screw hull form) the correction thus found can be approximated by following formula:

$$10^3 \cdot \Delta C_{R,bulb} = -0.2 - 1.1 \cdot \text{Fn (see Fig. C2)}$$

For Ro-Ro ships with twin-skeg hull form (twin screw propulsion) the correction thus found can be approximated by following formula:

$$10^3 \cdot \Delta C_{R,bulb} = 0.52 - 2.6 \cdot Fn$$
 (see Fig. C3)

For both hull forms the Froude number is based on the waterline length of the ship.

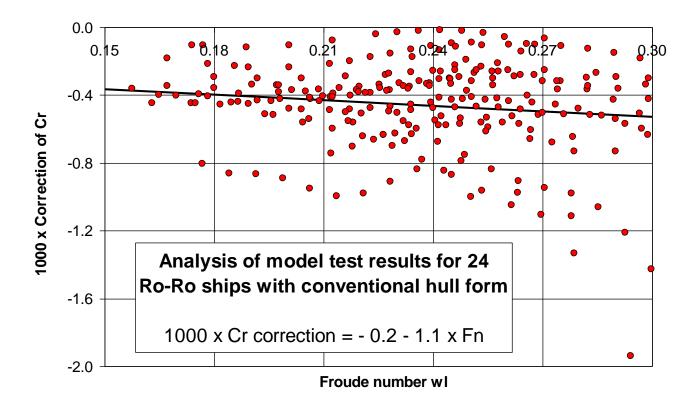


Fig. C2 The bulb correction for the residuary resistance coefficient for conventional hull forms

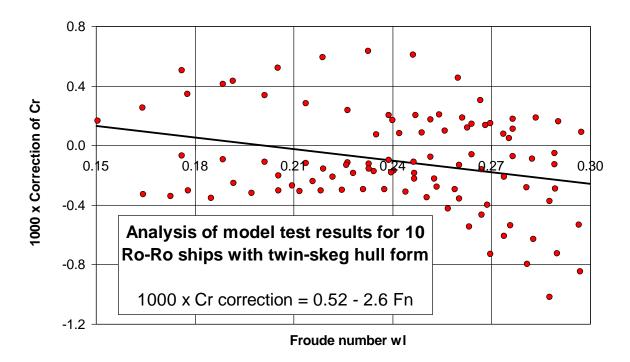


Fig. C3 The bulb correction for the residuary resistance coefficient for twin-skeg hull forms

The results of two model tests with ships with and without bulbous bow are plotted in Fig. C4. It is seen that the bulbous bow correction for conventional Ro-Ro ships found for use in the modified "Ship Resistance" method are in line with the average level found by direct model tests, where the influence of the bulbous bow has been investigated.

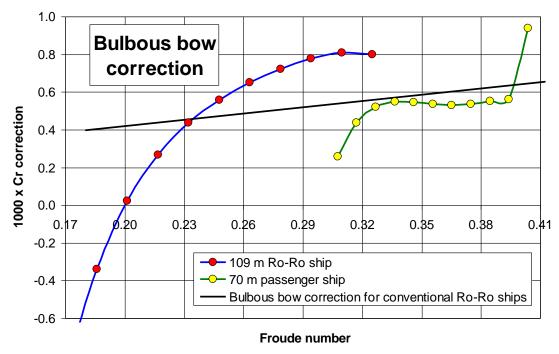


Fig. C4 Reduction of total resistance coefficient due to the influence of a bulbous bow. Found by model tests for two ships which have been tested with and without bulbous bow.

# **Appendix D - Propeller diameter**

The propeller diameter shall be as large as possible to obtain the highest efficiency. But in order to avoid cavitation and air suction, the diameter is restricted by the draught. In this appendix expressions for the propeller diameter as function of the maximum draught are given and documented by relevant statistical data in Fig. D1 and D2 based on data from ShipPax data base and Significant Ships (1990 – 2014).

Single screw Ro-Ro ships (cargo and pass. ships):  $D_{prop} = 0.56 \cdot max. draught + 1.07$ 

Twin screw Ro-Ro cargo ships:  $D_{prop} = 0.71 \cdot max. draught - 0.26$ 

Twin screw Ro-Ro passenger ships:  $D_{prop} = 0.85 \cdot max. draught - 0.69$ 

It is seen that the scatter of diameter to draught ratio is rather large (0.45 - 0.85) however with a majority of ships in the range between 0.65 and 0.75. The average value of Dprop/draught is 0.72 for single screw ships and twin screw passenger ships and 0.67 for twin screw cargo ships.

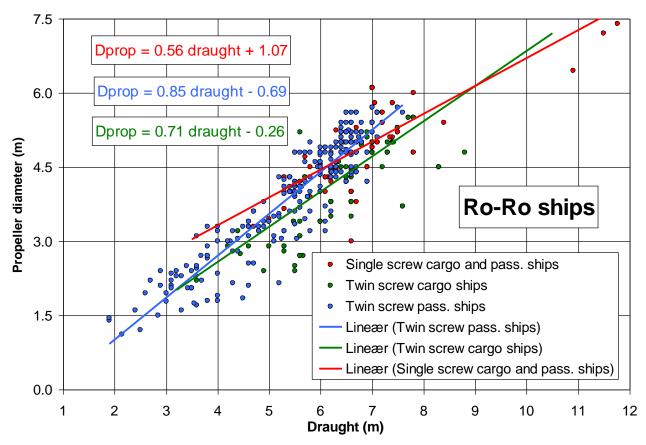


Fig. D1 Propeller diameter as function of maximum draught

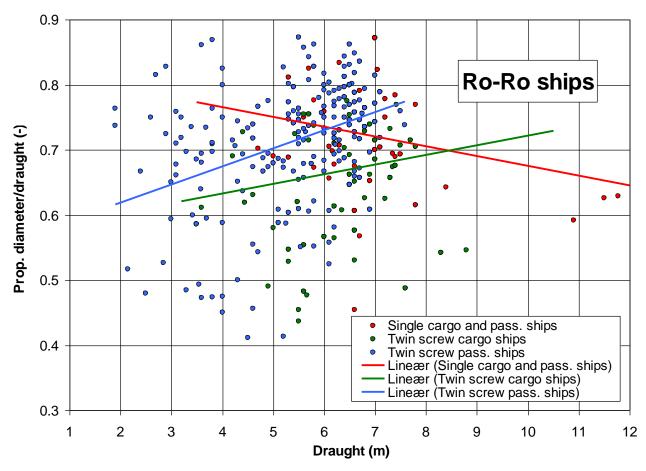
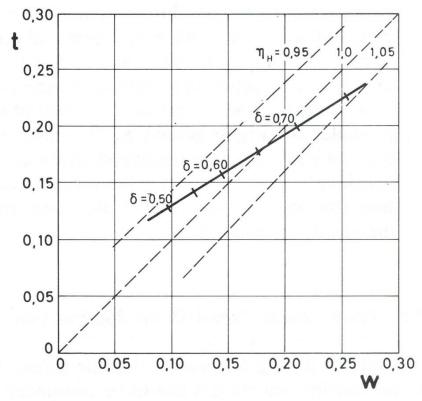


Fig. D2 Non dimensional propeller diameter (diameter/draught) as function of maximum draught

## Appendix E – Wake fraction and thrust deduction fraction for twin-skeg Ro-Ro ships

For conventional twin screw ships the wake fraction and thrust deduction fraction are calculated according to formulas based on Harvald [Harvald 1983, Figure 6.5.8]:

$$w = 1.133 \cdot C_B^2 - 0.797 \cdot C_B + 0.215$$
$$t = 0.0665 + 0.62833 \cdot w$$



**Figure 6.5.8.** Relationship among the thrust deduction fraction, the wake fraction, and the hull efficiency for twin-screw ships having normal form and D/L = 0.03.

For twin-skeg ships the wake fraction will be higher due to the skeg in front of each propeller. Based on analysis of 12 model test results with twin-skeg Ro-Ro ships (Fig. E1 and E2) following equations have been established for calculation of the wake fraction and the thrust deduction fraction of twin-skeg vessels as function of the water line block coefficient  $C_B$  for the wake fraction:

$$w = 0.7 \cdot C_B - 0.2$$

$$t = 0.19$$

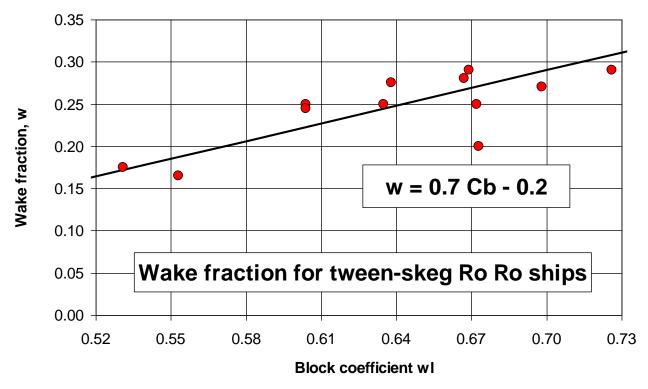


Fig. E1 Wake fraction, w, found by model tests for twin-skeg Ro-Ro ships

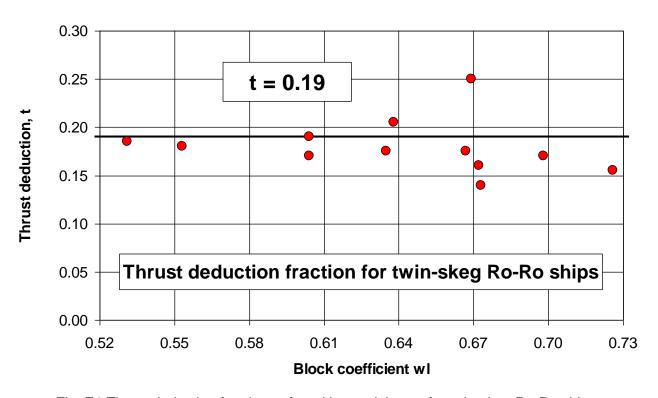


Fig. E1 Thrust deduction fraction, t, found by model tests for twin-skeg Ro-Ro ships

## **Hull efficiency**

The resulting hull efficiency has also been analyzed (Fig. G3). A relatively good agreement between the calculated efficiency and the measured hull efficiency is seen with a maximum deviation of approximately plus/minus 5 %

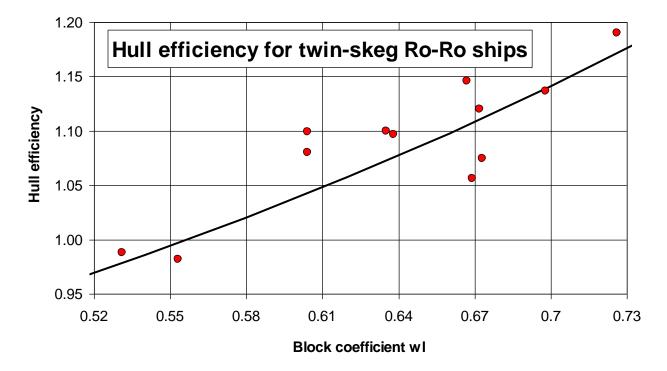
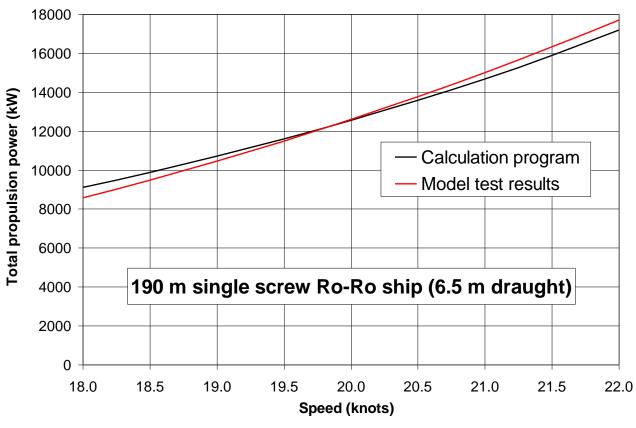
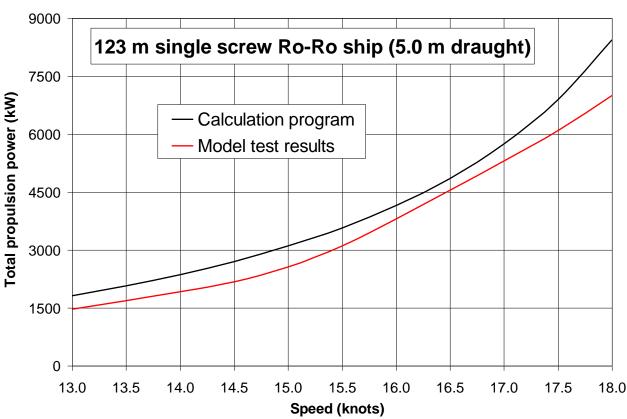
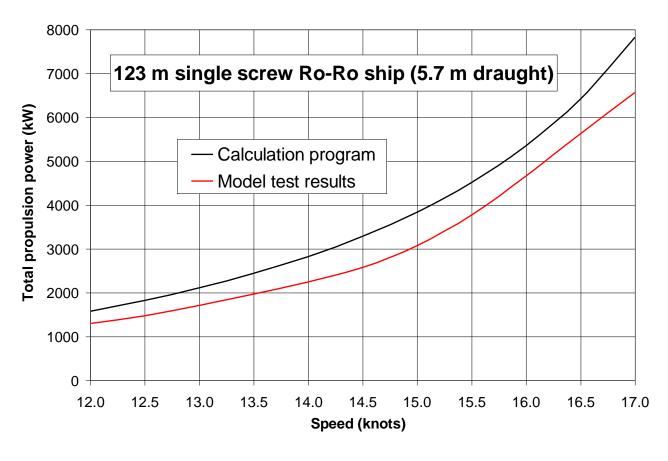


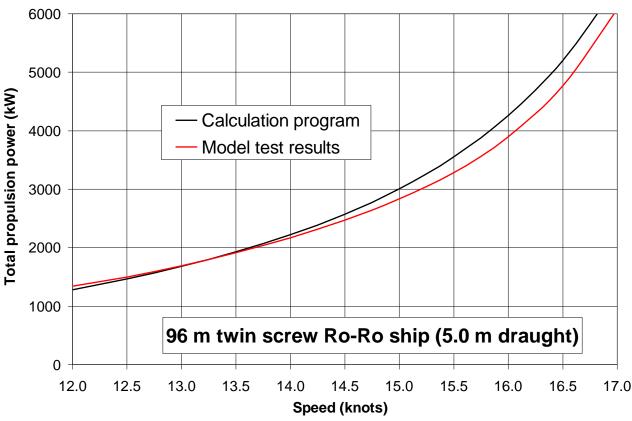
Fig. E3 Hull efficiency found by model tests for twin-skeg Ro-Ro ships compared with the values calculated by developed empirical formulas for wake fraction and thrust deduction fraction

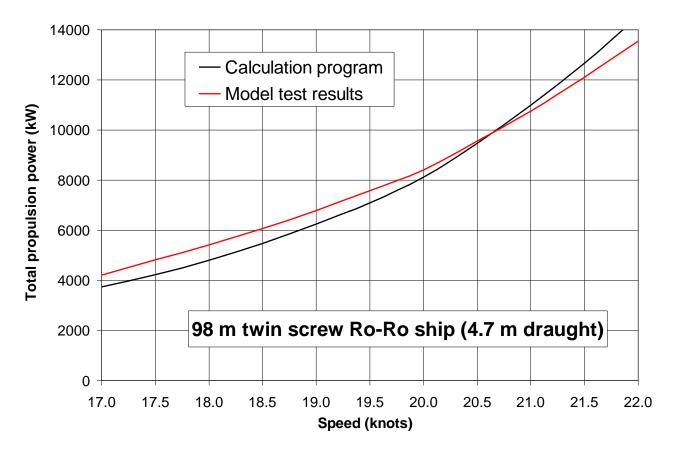
## Appendix F - Test calculations of propulsion power

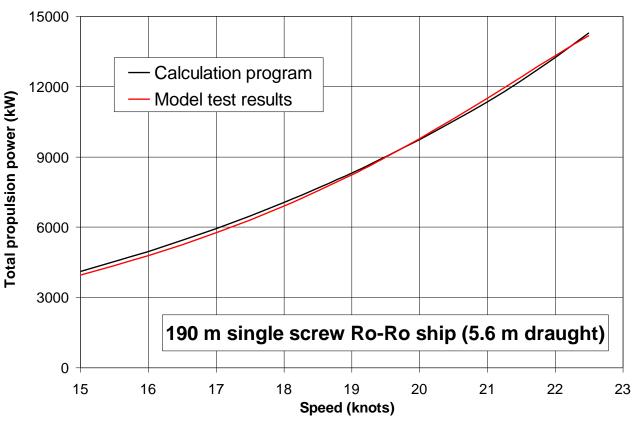


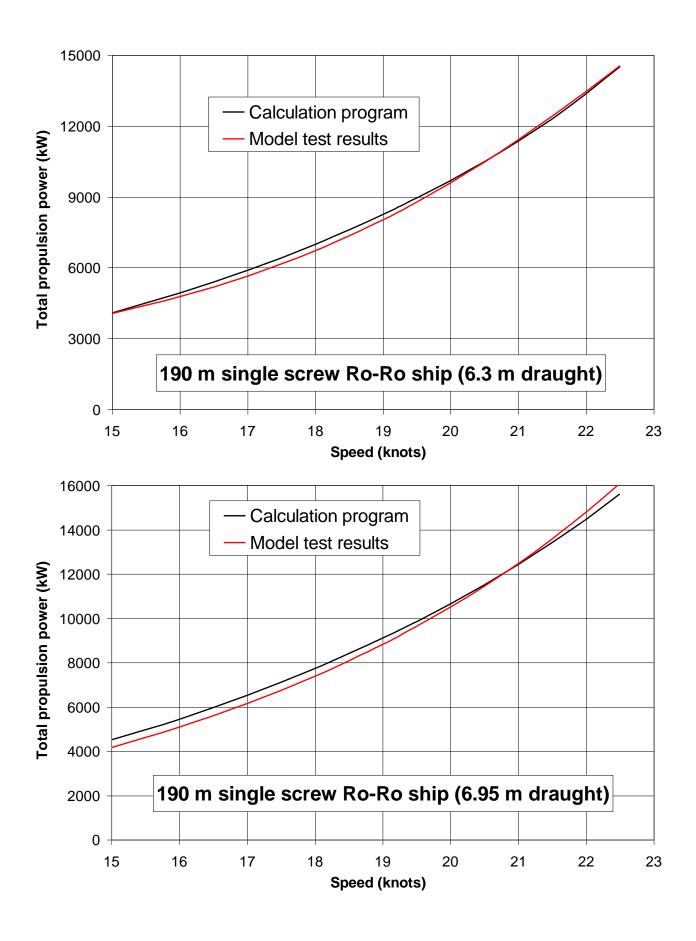


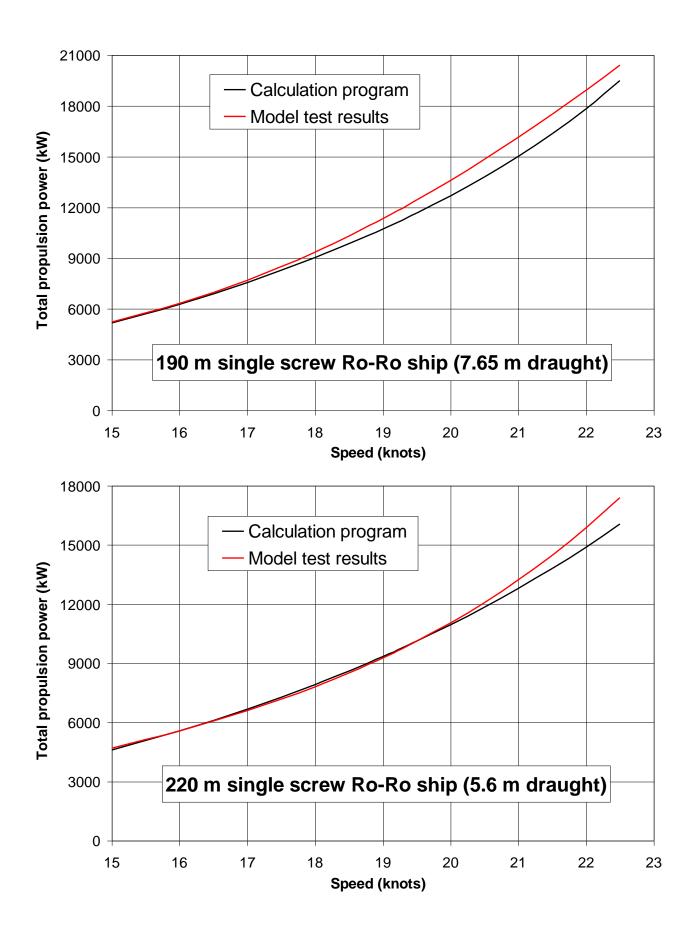


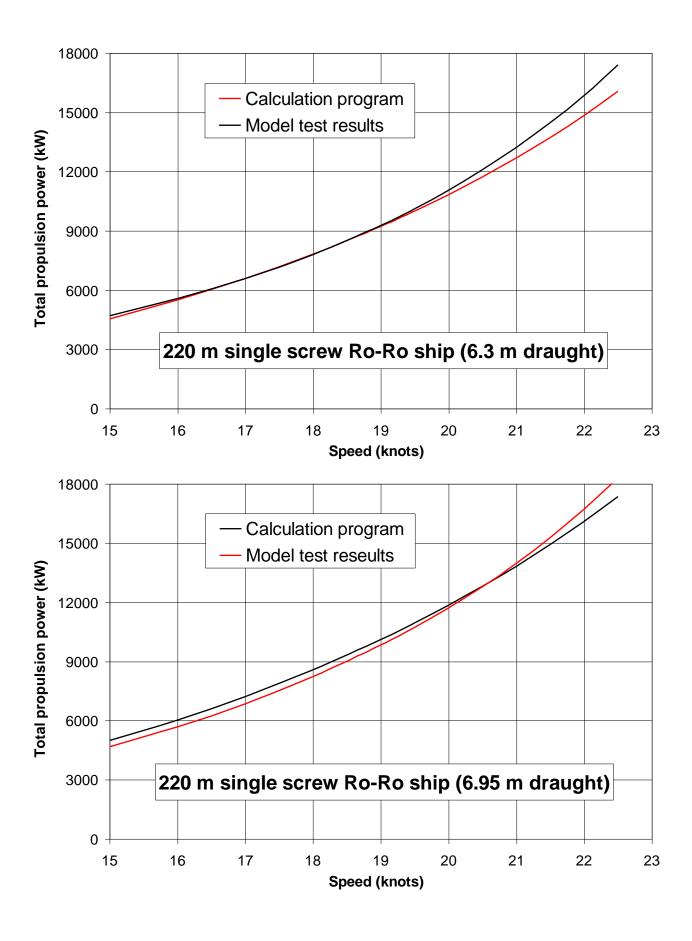


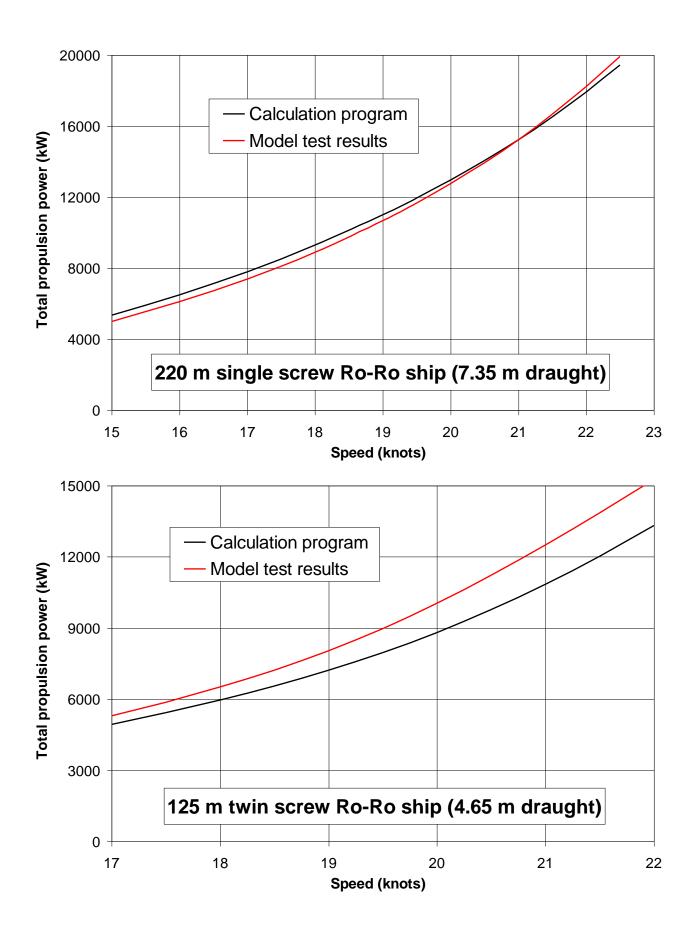


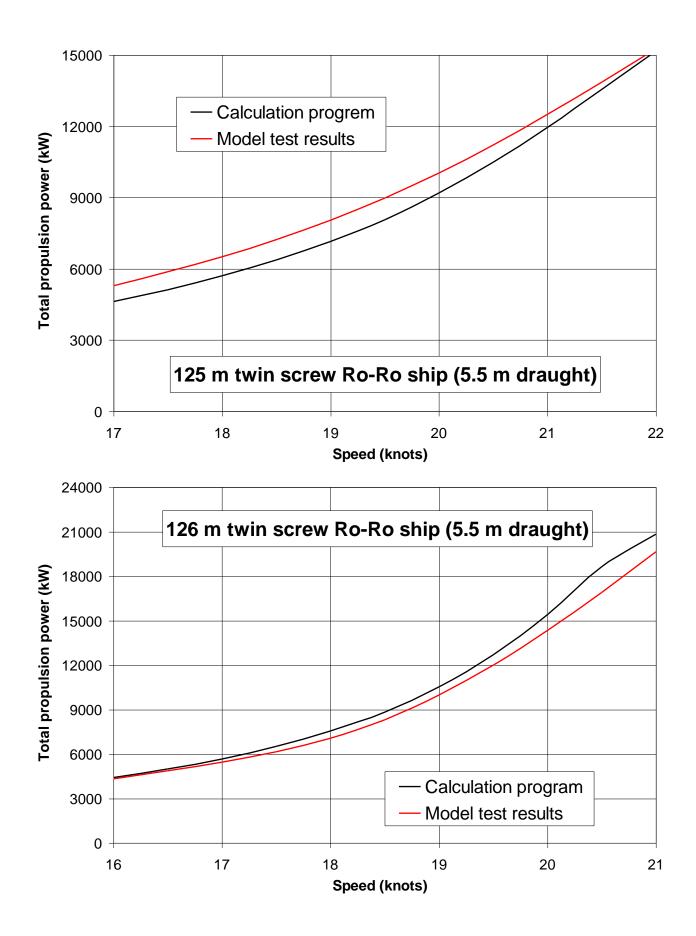


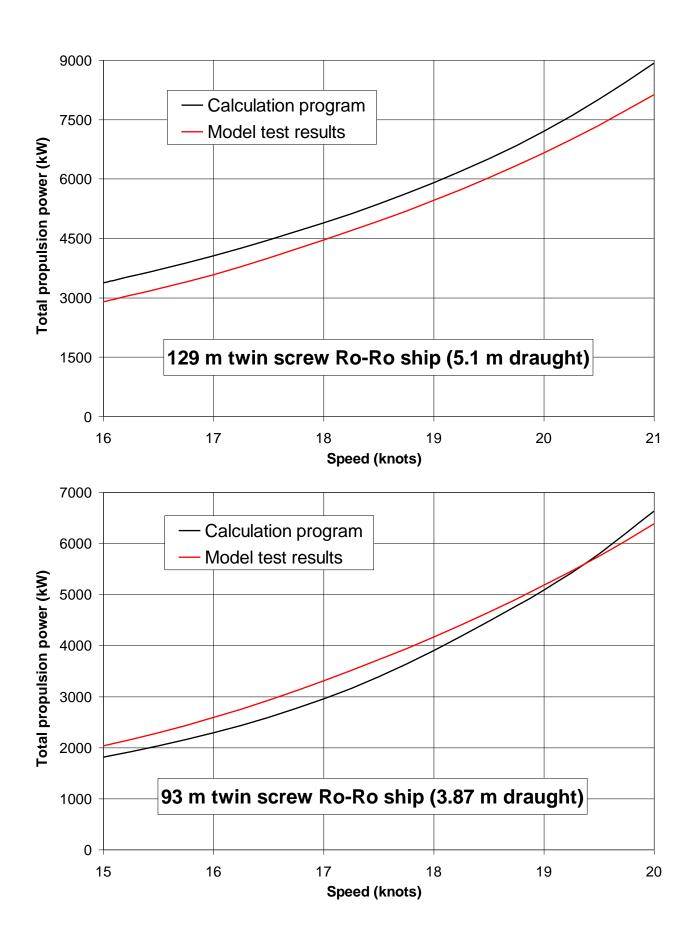


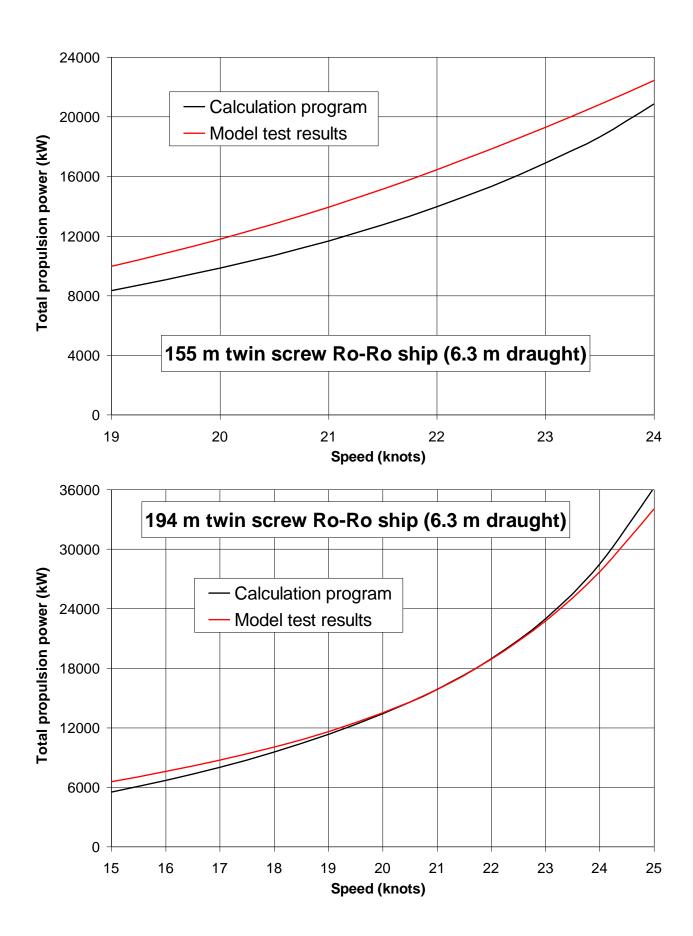


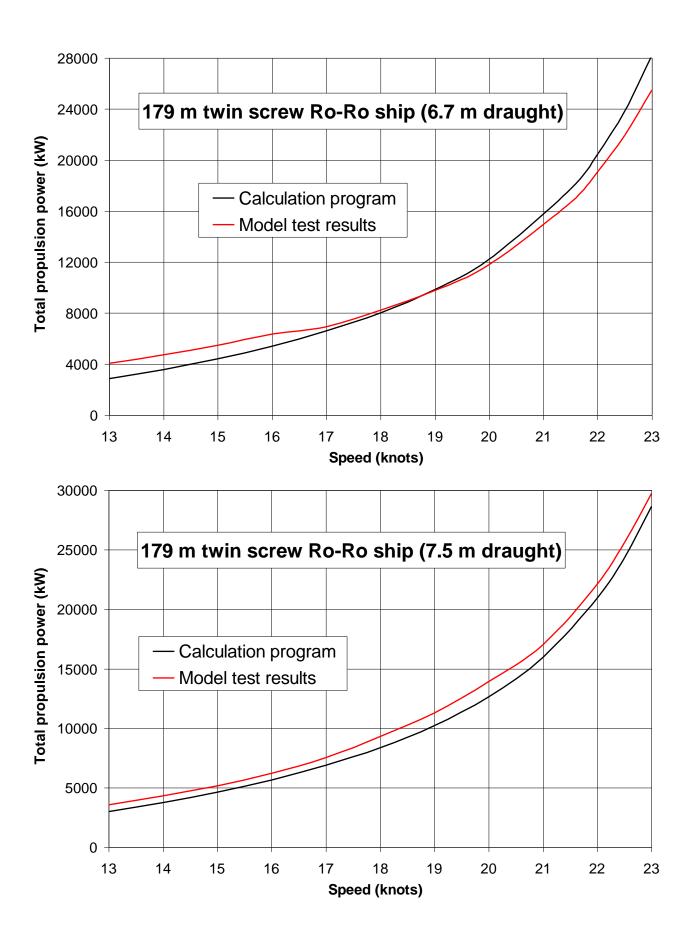


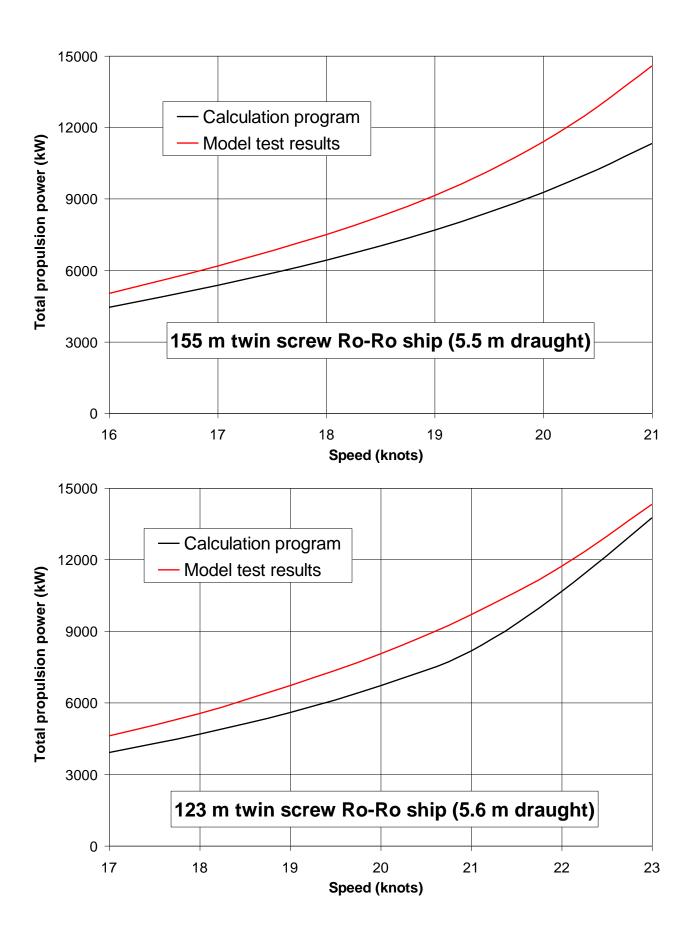


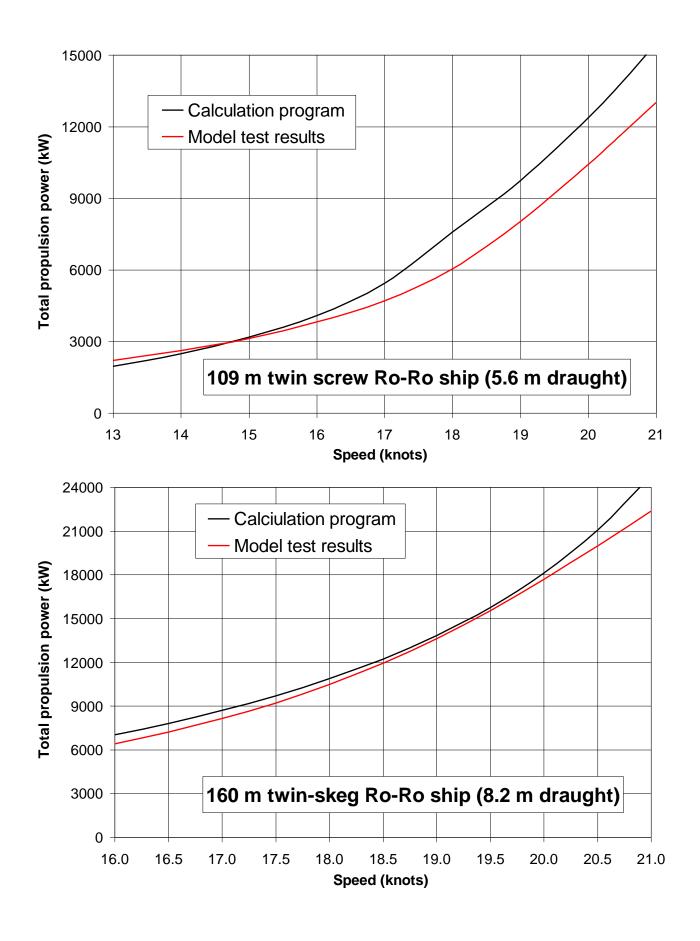


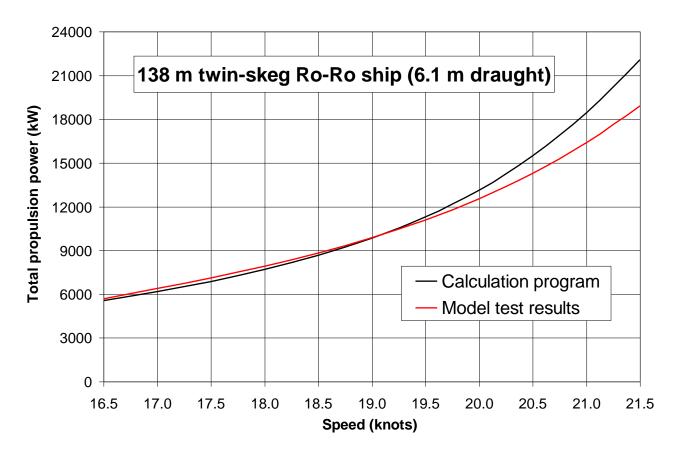


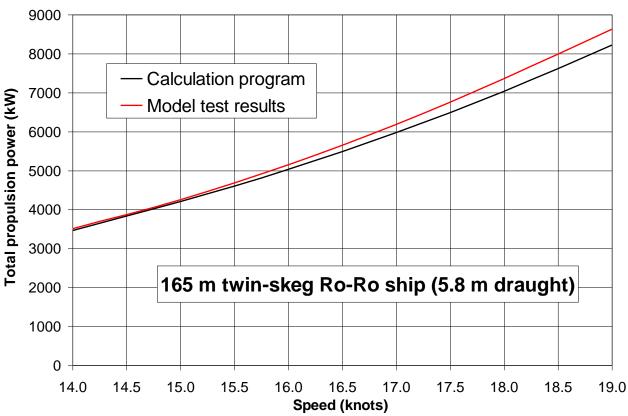


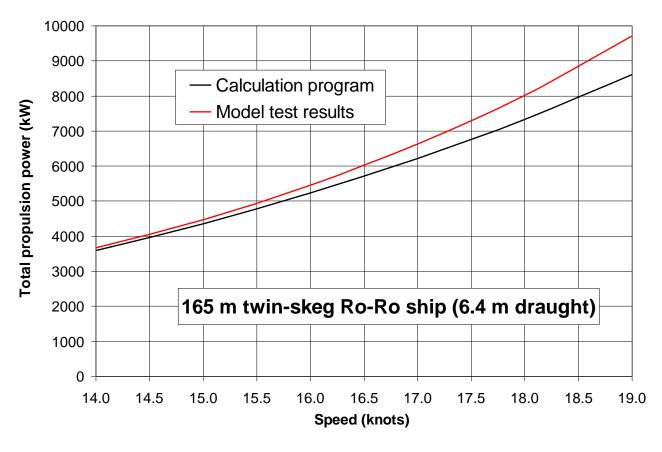


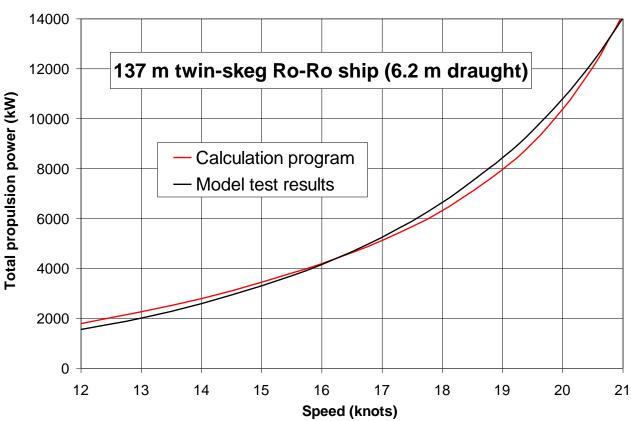


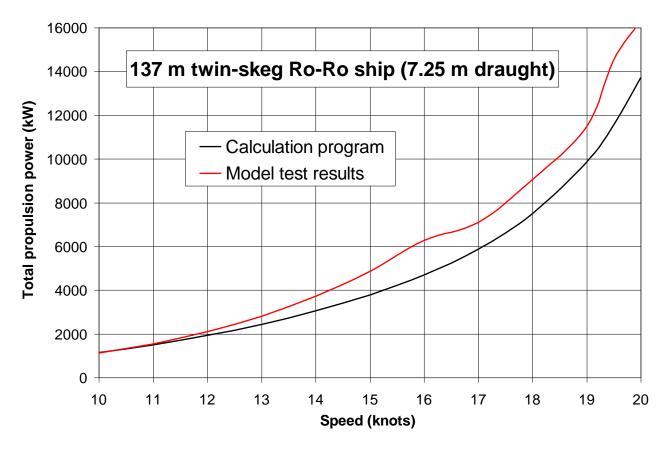


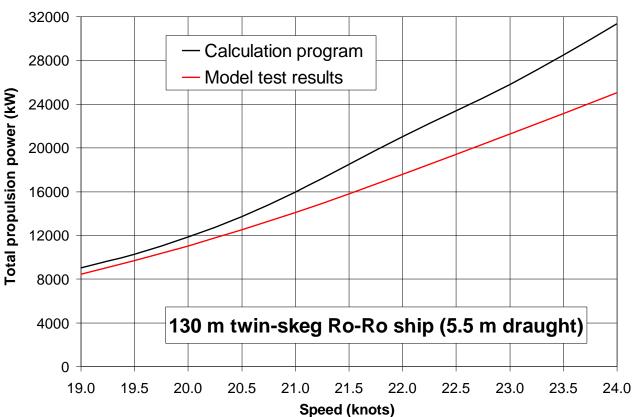


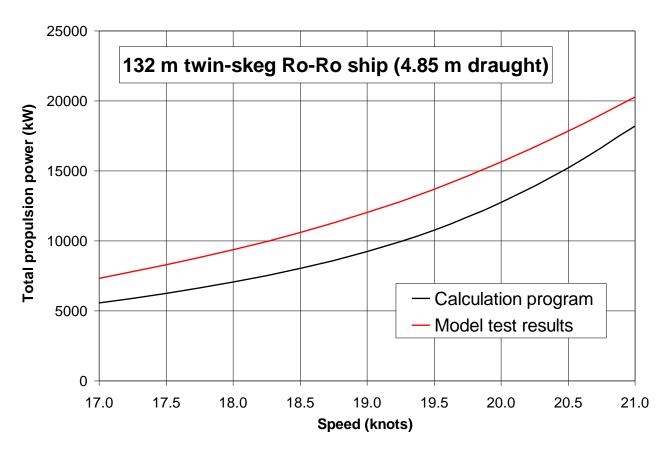


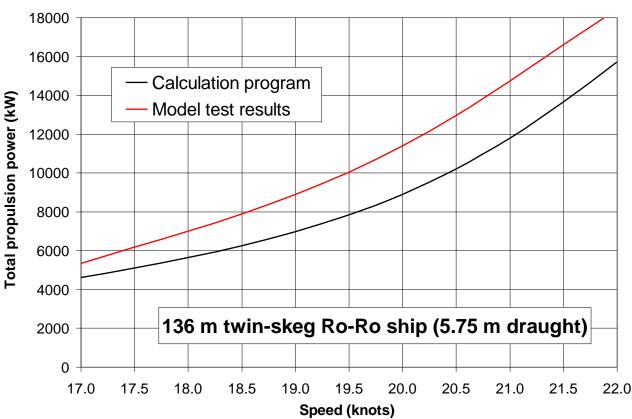


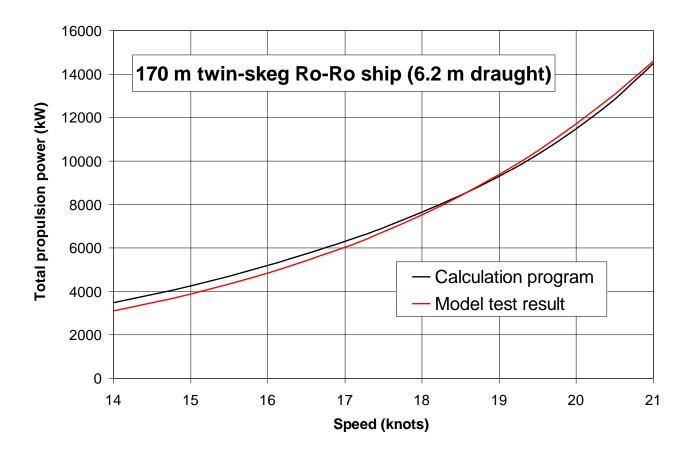












## Appendix G – Cr diagrams according to Guldhammer and Harvald

